Mr. Crab's Bootstrap

Roberto Baviera, Politecnico di Milano & RondPoint

join work with Alessandro Cassaro

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MR. CRAB'S
BOOTSTRAP

Outline

1. Discounting curve (e.g. EONIA)

2. Estimation curve (e.g. Euribor 3m, Euribor 6m)

3. Numerical example: Euro curve

4. An application

1. Discounting curve (e.g. EONIA)

   - ✓ What is needed in the construction
   - ✓ No "additional" interpolation

2. Estimation curve (e.g. Euribor 3m, Euribor 6m)

3. Numerical example: Euro curve

4. An application

Discounting curve: EONIA

OIS maturity lower or equal 1y (only 1 payment date)

$$P^D(t_0, t_e) = \frac{1}{1 + \delta(t_0, t_e) R^{OIS}(t_0, t_e)}.$$

OIS maturity > 1y (more payment dates)

$$P^D(t_0, t_i) = \frac{1 - R^{OIS}(t_0, t_i) \sum_{k=1}^{i-1} \delta_k P^D(t_0, t_k)}{1 + \delta_i R^{OIS}(t_0, t_i)}.$$

See e.g. Ametrano & Bianchetti (2009, 2010, 2011)

Discounting curve: EONIA

Derivatives considered:

- ✓ OIS $\leq$ 1y: 1w-3w, 1m-12m;
- ✓ OIS > 1y: 15m, 18m, 21m (with short stub in advance); 2y-30y;

Remarks:

- ✓ EONIA rates construction avoids to consider forward (daily) EONIA rates. Only OIS (Overnight Indexed Swap) rates are needed in the bootstrap.
- ✓ No "additional" interpolation in the bootstrap construction.

1. Discounting curve (e.g. EONIA)

2. Estimation curve (e.g. Euribor 3m, Euribor 6m)

   - ✓ Moving as Mr. Crab
   - ✓ Modeling framework
   - ✓ STIR Future convexity adjustment

3. Numerical example: Euro curve

4. An application

Estimation curve: Euribor 3m

Target:

- ✓ Obtaining a homogeneous bootstrap using only derivatives with Euribor 3m as underlying

Idea:

Moving Forward & Backwards (i.e. as a Crab)

Ingredients in the bootstrap:

- ✓ Euribor 3m fixing (previous day fixing before 11:00 C.E.T.)
- ✓ FRA 3x6, 2x5, 1x4
- ✓ STIR Futures
- ✓ Swap vs 3m: 2y-30y

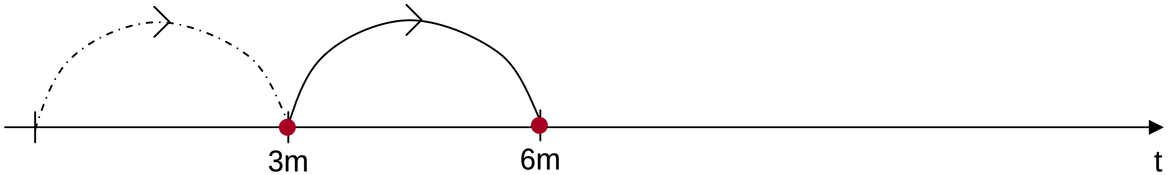
Idea for the *Estimation curve* bootstrap: Moving as a Crab (1)

… moving forward (Depo & 1st FRA)

Depo (Euribor 3m Fixing):

● : value obtained from market data

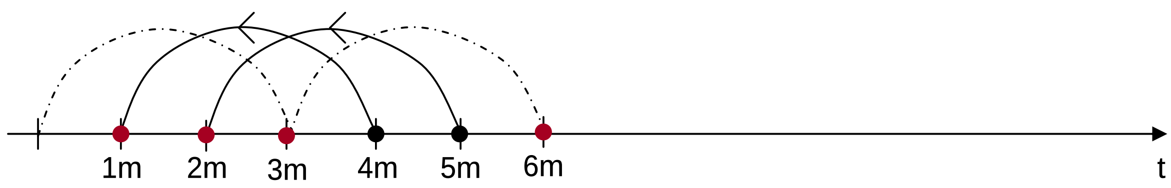
3x6 FRA



… moving backward (other FRAs)

● : value obtained from market data
● : value from interpolation

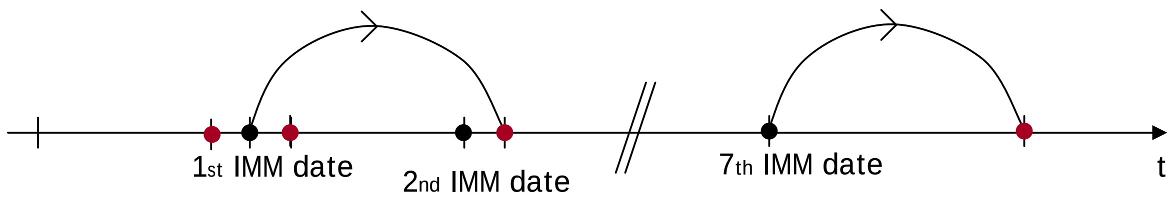
FRA 2x5 e 1x4



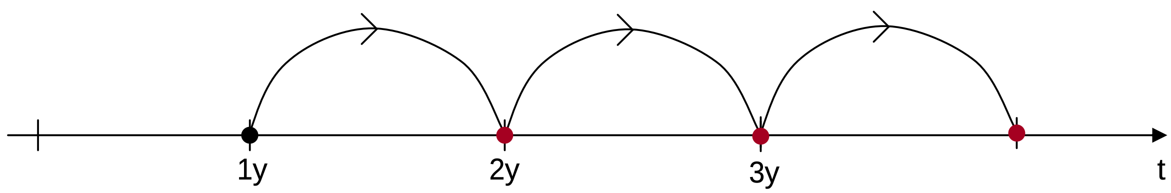
… and forward again (STIR futures & Swaps)

● : value obtained from market data
● : value from interpolation

STIR Futures



Swaps



Mr. Crab's bootstrap

Estimation curve: Modeling framework (1)

Main definitions:

- ✓ Forward Rate

$$P^D(t_0, t_e)F(t_0; t_s, t_e) \equiv E[D(t_0, t_e)L(t_s, t_e)|\mathcal{F}_0]$$

- ✓ Estimation curve

$$P(t_0; t_s, t_e) \equiv \frac{P(t_0, t_e)}{P(t_0, t_s)} \equiv \frac{1}{1 + \delta(t_s, t_e)F(t_0; t_s, t_e)}$$

- ✓ Curve Ratio

$$\beta_t(t_s, t_e) \equiv \frac{P^D(t; t_s, t_e)}{P(t; t_s, t_e)}$$

Henrard (2010)

Main hypothesis:

- ✓ Hypothesis SI

$$\beta_t(t_s, t_e) \not\Leftrightarrow P^D(t; t_s, t_e)$$

Independent

Main consequence:

- ✓ FRA & Forward rates are equivalent

$$F(t_0; t_s, t_e) = F^{FRA}(t_0; t_s, t_e)$$

Henrard (2010)

Main relation:

$$\sum_{k=1}^{f\times i} w_k \, F_k(t_0) = S(t_0,t_i) \sum_{k=1}^{i} \delta_k \, P^D(t_0,t_k) \qquad i \geq 2$$

where

$$w_k \equiv \tilde{\delta}_k \, P^D(t_0,\tilde{t}_k)$$

and

$$f = 4$$

⇨ Bootstrap via an interpolation rule

GHJM within hypothesis SI

- ✓ Discounting curve

$$P^D(t;t_s,t_e) = P^D(t_0;t_s,t_e) \exp\left\{ -\frac{1}{2}\int_{t_0}^{t} [\nu^D(u,t_e)^2 - \nu^D(u,t_s)^2] du - \int_{t_0}^{t} [\nu^D(u,t_e) - \nu^D(u,t_s)] \cdot dW_u \right\},$$

- ✓ Curve Ratio

$$\beta_t(t_s,t_e) = \beta_0(t_s,t_e) \exp\left\{ -\frac{1}{2}\int_{t_0}^{t} [\eta(u,t_e) - \eta(u,t_s)]^2 du + \int_{t_0}^{t} [\eta(u,t_e) - \eta(u,t_s)] \cdot dW_u \right\}.$$

with $\begin{cases} \nu^D(u,t), \eta(u,t), W_u \in \Re^M \\ dW_i(u)\, dW_j(u) = \rho_{ij}\, du \end{cases}$ and $\nu^D(u,t) \cdot \rho\eta(u,t) = 0$

Main consequences:

✓ Forward Rate *vs* Future Rate

$$F(t_0; t_s, t_e) = \frac{1}{1+\tilde{\gamma}_1}\left\{F^{fut}(t_0; t_s, t_e) - \frac{\tilde{\gamma}_1}{\delta(t_s, t_e)}\right\}$$

where the Convexity Adjustment is equal to:

$$\tilde{\gamma}_1 = \exp\left\{\int_{t_0}^{t_s} \nu^D(u, t_e) \cdot \rho(\nu^D(u, t_e) - \nu^D(u, t_s))du\right\} - 1$$

Oss. 1: $\text{Convexity} \leftarrow \nu^D(u, t)$

✓ Black like formula for STIR future's Options (see Theorem 3)

Oss. 2: $\text{Price} \leftarrow \nu^D(u, t), \eta(u, t)$

Ingredients in the volatility calibration:

- ✓ Options' STIR Futures

Two limit hypotheses within SI hypothesis:

- ✓ S0 hypothesis: Curve Ratio is constant

$$\eta(t,T) = 0$$

- ✓ S1 hypothesis: Discounting Curve is constant

$$\nu^D(t,T) = 0$$

... and then no convexity adjustment $\tilde{\gamma}_1$

STIR Volatilities & Convexity adjustments (13 Sep 2012 Example):

	no adjustment (hypothesis **S1**)		*deterministic ratio* (hypothesis **S0**)	
Expiry date	$\sigma_i^{mkt}(\%)$	$\tilde{\gamma}_1/\delta(t_s,t_e)$ (bps)	$\sigma_i^{mkt}(\%)$	$\tilde{\gamma}_1/\delta(t_s,t_e)$ (bps)
19-Dec-12	-	0.00	-	0.00
19-Mar-13	0.23	0.00	0.23	0.10
20-Jun-13	0.28	0.00	0.27	0.29
19-Sep-13	0.32	0.00	0.32	0.63
18-Dec-13	0.35	0.00	0.34	1.15
18-Mar-14	0.34	0.00	0.33	1.94
19-Jun-14	0.40	0.00	0.38	3.14

Estimation curve: Euribor 6m

Target:

- ✓ Obtaining a homogeneous bootstrap using only derivatives with Euribor 6m as underlying

Idea:

Moving Forward & Backwards (i.e. as a Crab)

Ingredients in the bootstrap:

- ✓ Euribor 6m fixing (previous day fixing before 11:00 C.E.T.)
- ✓ FRA (1) 6x12, 5x11, 4x10, 3x9, 2x8, 1x7 : Backward
- ✓ FRA (2) 7x13, 8x14, …, 12x18 : Forward
- ✓ Swap vs 6m: 2y-30y

1. Discounting curve (e.g. EONIA)

2. Estimation curve (e.g. Euribor 3m, Euribor 6m)

3. Numerical example: Euro curve

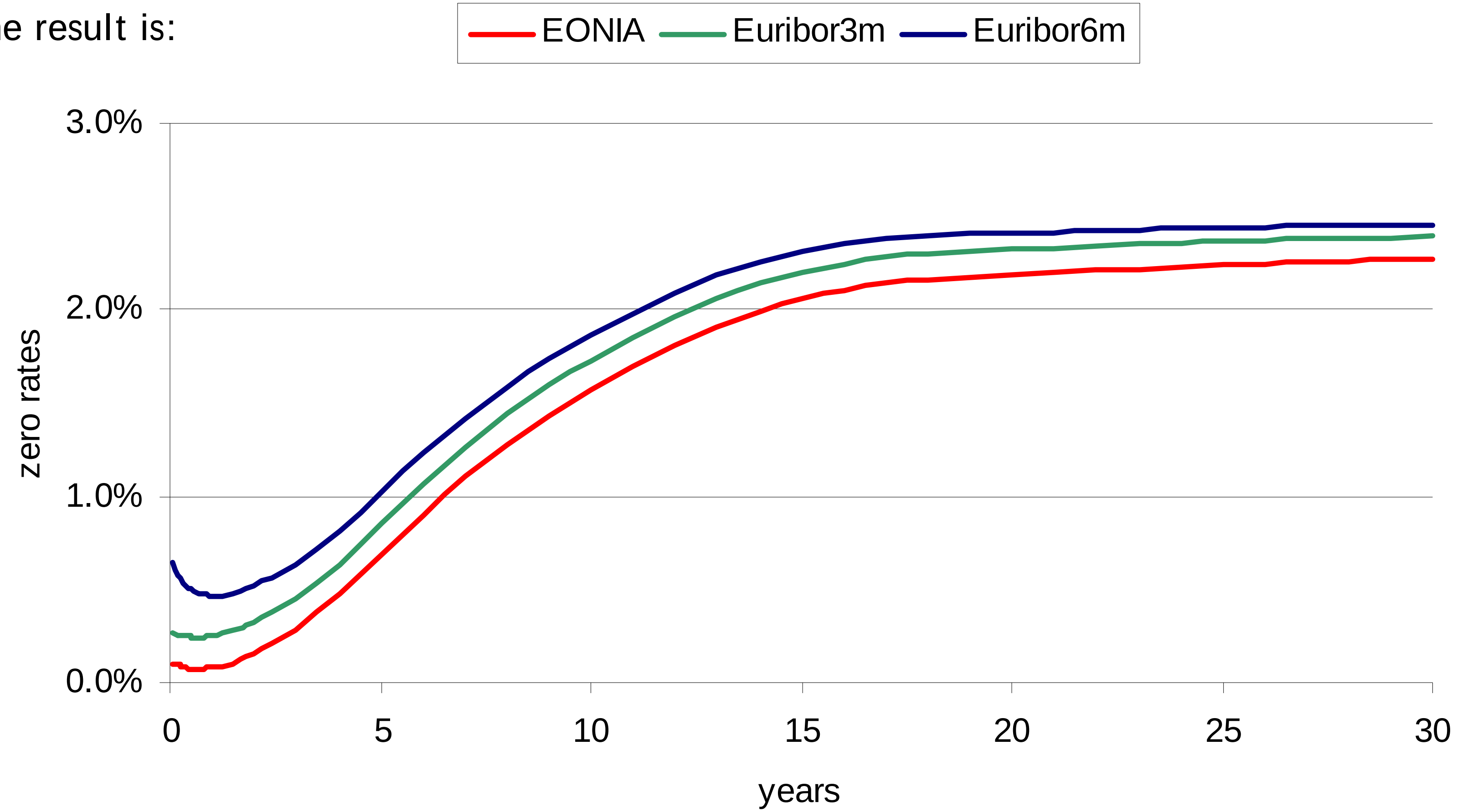
   - ✓ Bootstrap
   - ✓ Convexity adjustment
   - ✓ Comparison with Bloomberg
   - ✓ Indipendence from “additional” interpolation
   - ✓ Turn of Year effect

4. An application

Example: Euro curves 13 Sep 2012 at 16:18 C.E.T.

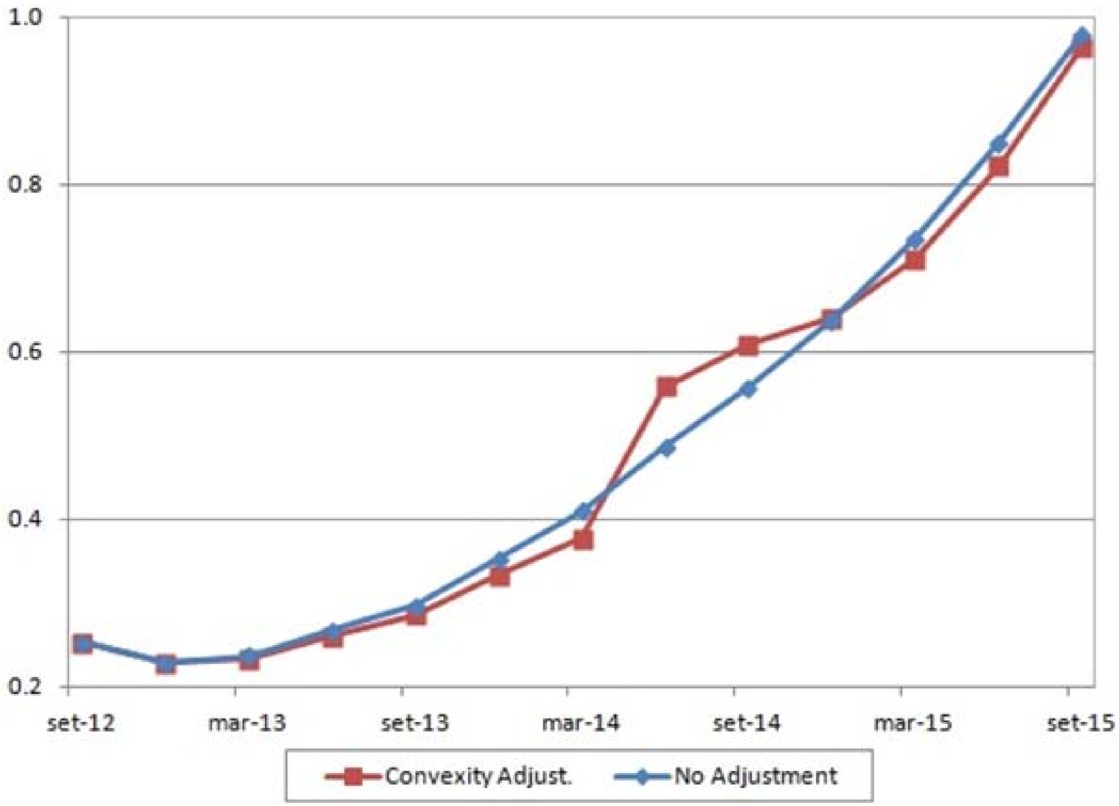
Interpolation rule: linear on zero-rates.

The result is:



... with no smoothing!

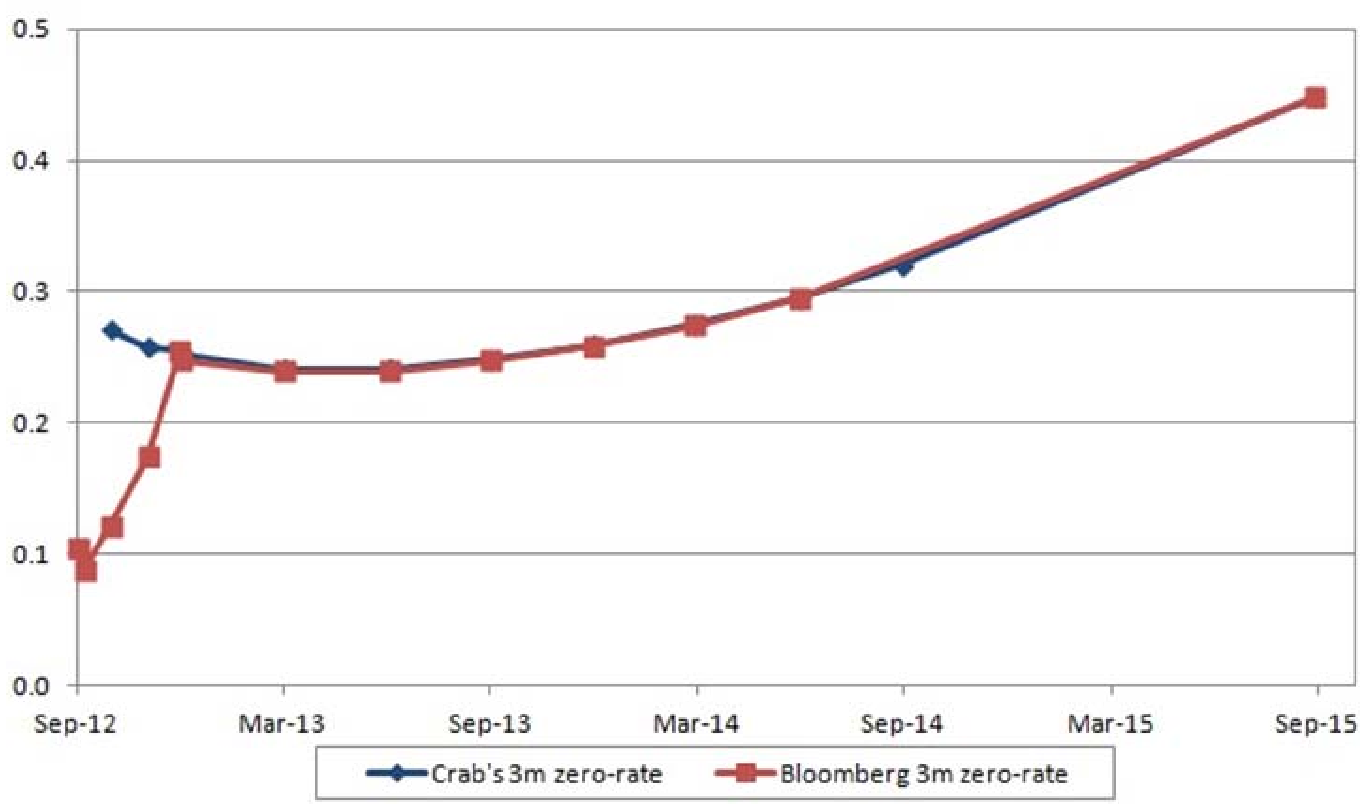
S1 vs S0 hypothesis:



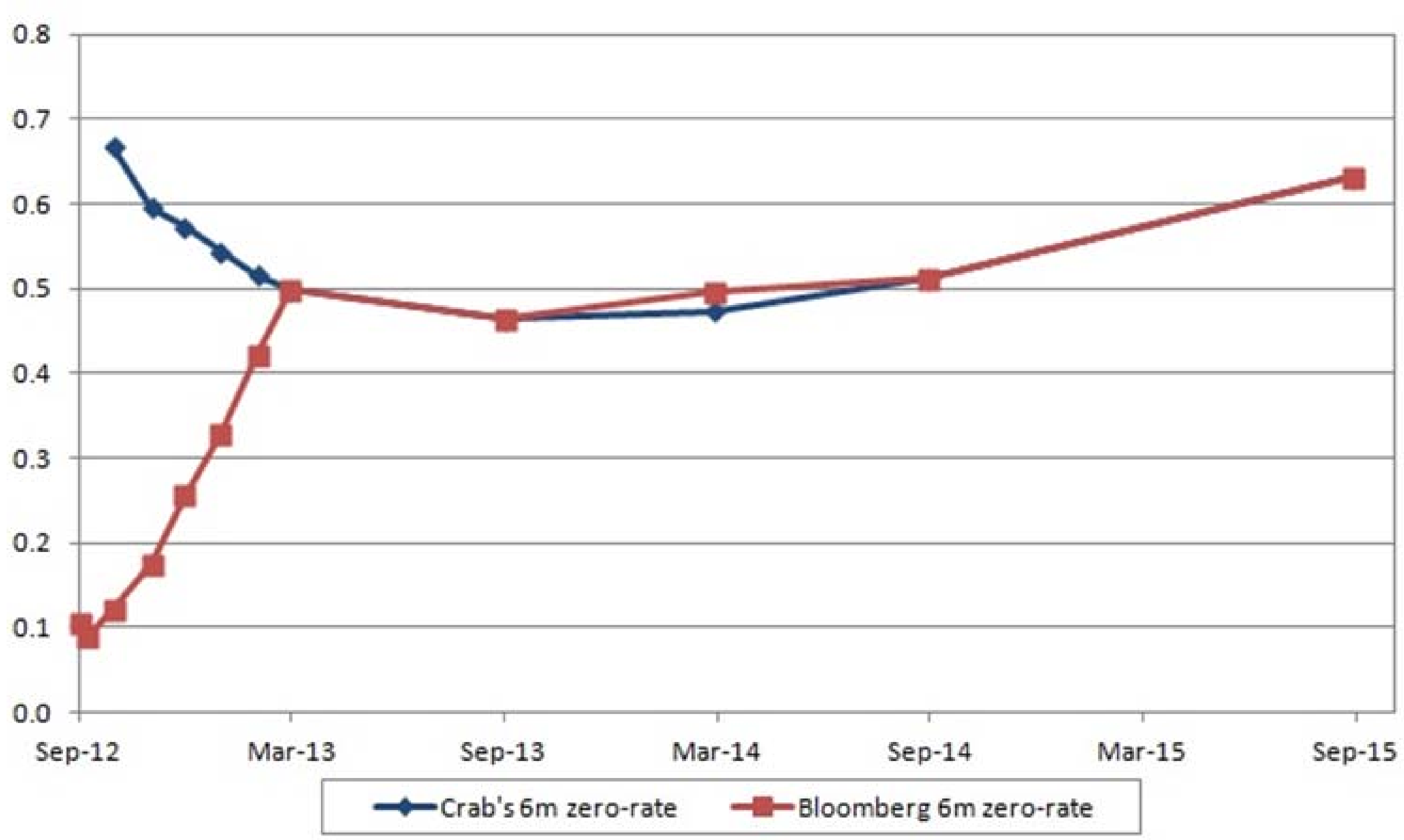
	Market (bps)	Computed S1 (bps)	Computed S0 (bps)
18m swap	27.4	27.5	26.8

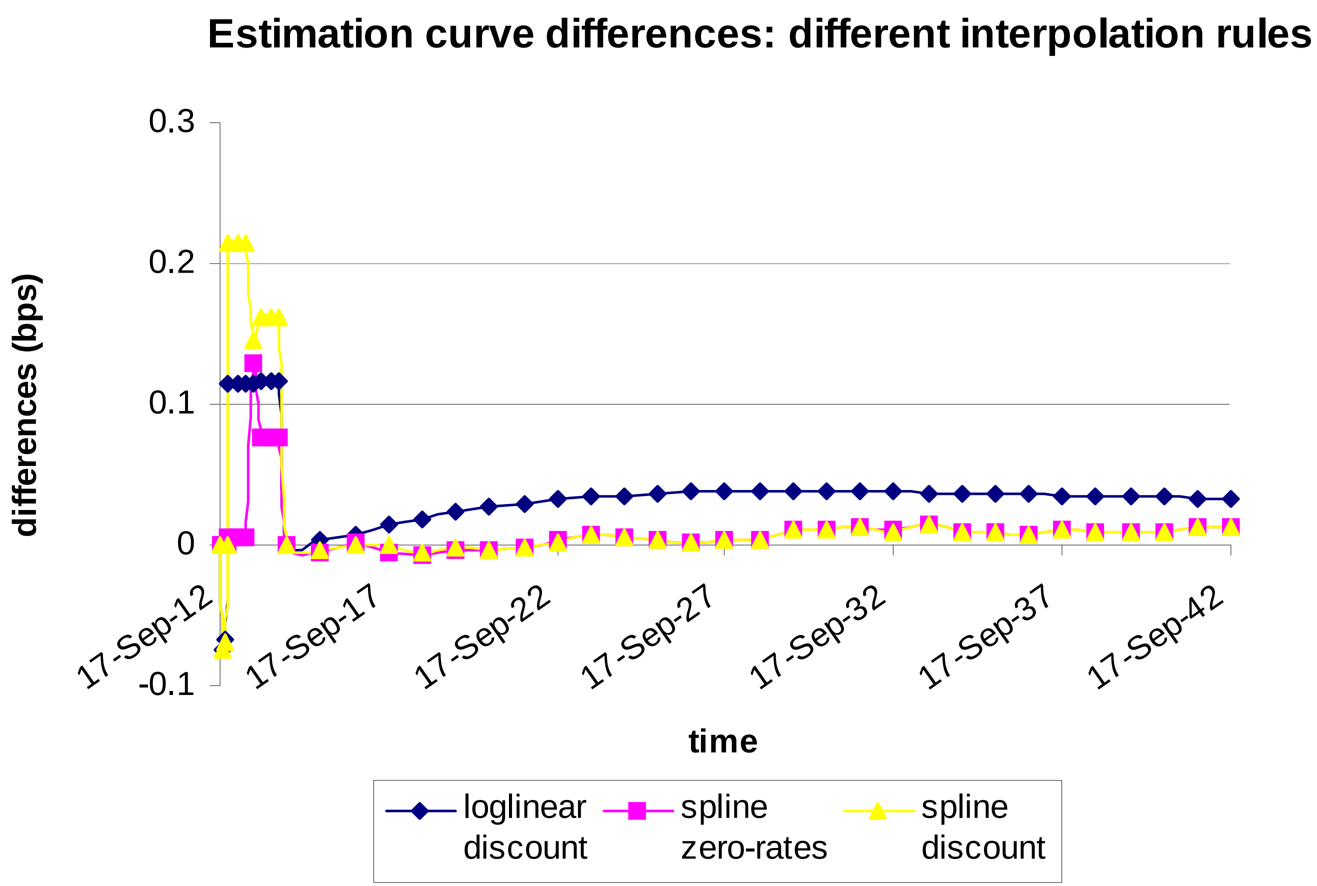
Calibration on STIR options

Euribor 3m. Crab *vs* Bloomberg: zero rates



Euribor 6m. Crab *vs* Bloomberg: zero rates




Estimation curve differences: different interpolation rules
0.3
0.2
0.1
0
-0.1
differences (bps)
17-Sep-12
17-Sep-17
17-Sep-22
17-Sep-27
17-Sep-32
17-Sep-37
17-Sep-42
time
loglinear discount
spline zero-rates
spline discount

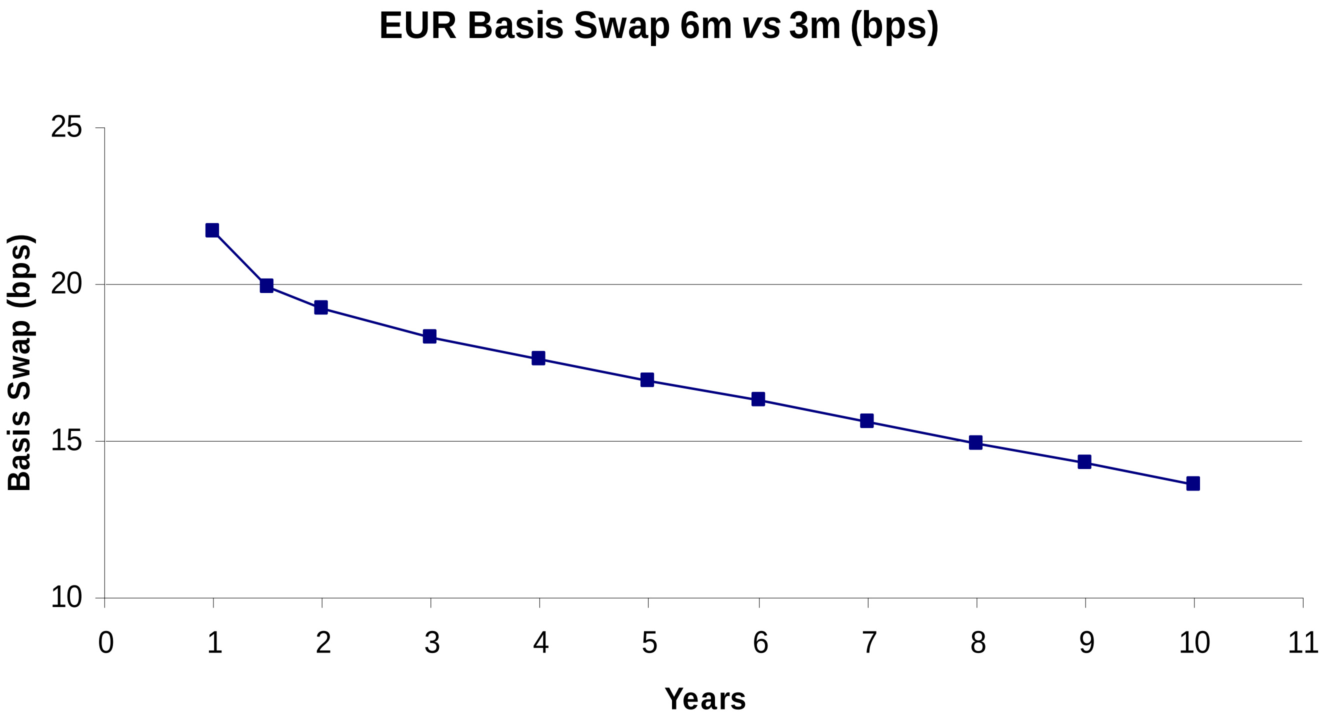
Mr. Crab's bootstrap

Turn of Year (ToY) effect

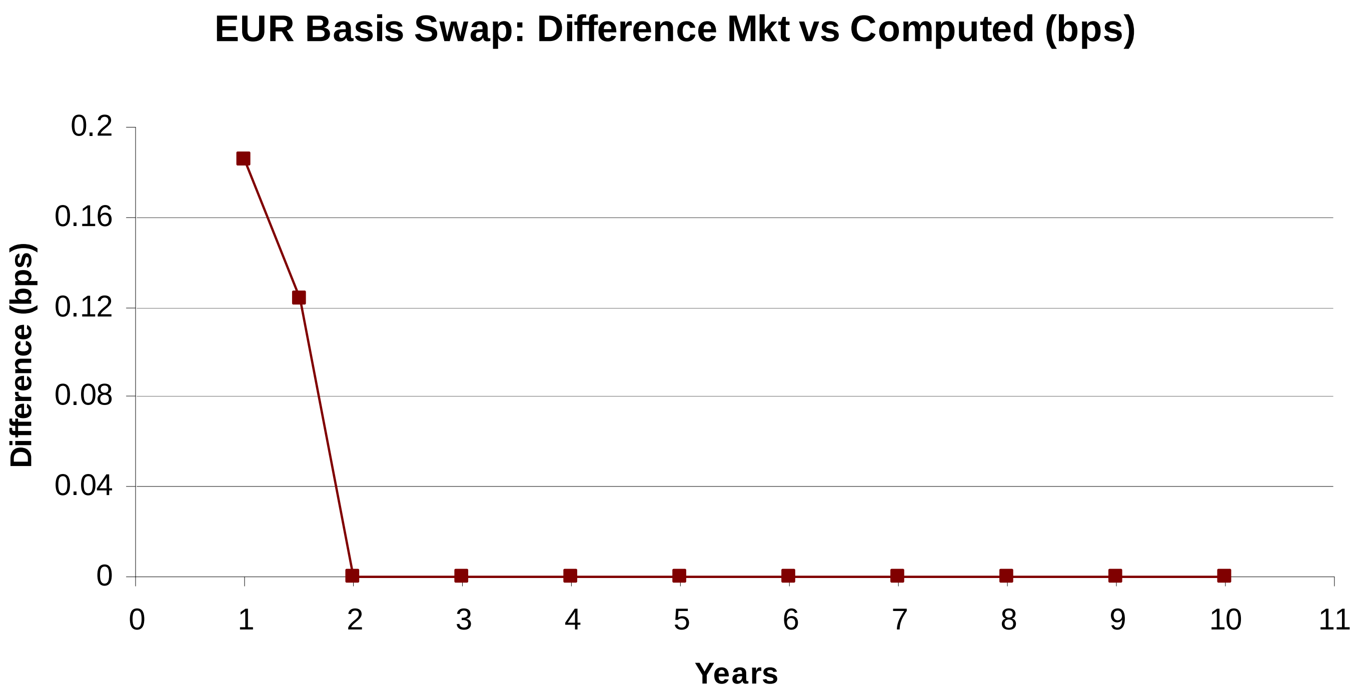
(similar results hold for USD)

1. Discounting curve (e.g. EONIA)
2. Estimation curve (e.g. Euribor 3m, Euribor 6m)
3. Numerical example: Euro curve
4. **An application**
   - ✓ Basis Swap

Basis risk: 6m vs 3m



Basis risk: 6m vs 3m



Conclusions

A coherent bootstrap technique for Eur, Usd IR Curves vs 3m and 6m

Main results:

- Negligible convexity adjustments (calibration via STIR Future options)
- Negligible interpolation rule impacts
- Negligible ToY effect

bibliography sketch

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TWO STEPS FORWARD, TWO STEPS BACKWARDS AND THEN FORWAR