International Journal of Theoretical and Applied Finance
Vol. 5, No. 6 (2002) 575–583

MOVING AVERAGES AND PRICE DYNAMICS

R. BAVIERA

Departement Finance & Economie, Groupe HEC,
Rue de la Liberation 1, F-78351 Jouy-en-Josas, France

M. PASQUINI

Istituto Nazionale di Fisica della Materia, I-67010 Coppito, L'Aquila, Italy
michele.pasquini@tin.it

J. RABOANARY

Institute Superieur Polytecnique de Madagascar, Lot G III 32 Bis,
Soamandrariny, Antananarivo, Madagascar

M. SERVA

Dipartimento di Matematica, Università dell'Aquila,
I-67010 Coppito, L'Aquila, Italy

Received 17 August 2001
Accepted 10 December 2001

We introduce a stochastic price model where, together with a random component, a moving average of logarithmic prices contributes to the price formation. The future price is linearly influenced by the difference between the moving average and the current price, together with a noise component. Our model is tested against financial datasets, showing an extremely good agreement with them.

Keywords: Price dynamics; moving average; Nasdaq index; foreign exchange markets.

1. Introduction

Moving averages are probably the most popular and elemental analysis tool in finance, widely known and applied both by professional and amateur traders. Such a large favour is due to their simplicity and intuitive meaning, which can help to understand the more or less hidden trend of an evolving dataset, filtering some of the noise.

Moving averages are often studied and taken into account in modeling price dynamics both in financial literature and in textbooks for market traders (technical analysis) [1–6].

In this letter we introduce a model for price dynamics where a moving average plays a central role (in the following with "price" we mean a more general class of financial objects, including share quotes, indices, exchange rates, and so on). We focus our attention on the logarithm of price, not the price itself, since the difference of two consecutive logarithmic prices corresponds to price return, being a universal measure of a price change, not affected by size factors.

The moving average is therefore computed with logarithmic prices, but we do not simply look a time window in the past, giving the same weight to each past quote. On the contrary, we take into account every quote in the past, but its weight decays exponentially with time [7]. This choice is suggested by several evidences that correlations between price changes rapidly go to zero [8, 9], while absolute price changes are long range correlated (see for example [10]).

In our price dynamics the difference between the logarithmic price and the moving average at a given time linearly influences the future price, together with noise. Let us stress that the moving average can either attract or repulse the future price, being this a peculiar feature of the market considered. In turns out that the logarithmic price plus its moving average evolves following a one-step Markov process.

We test our model analyzing three datasets:

(a) Nasdaq index: 4032 daily closes from 1st November 1984 to 25th September 2000.
(b) Italian Mibtel index: 1700 daily closes from 4th January 1994 to 2nd October 2000.
(c) 1998 US Dollar–Deutsche Mark (USD–DM) bid exchange rate: 1620853 high frequency data, that represent all worldwide 1998 bid quotes, made available by Olsen & Associated.

The average time difference between two consecutive data is about 20 seconds.

The results give a clear evidence that our model for price dynamics is fully consistent with all the above datasets.

2. Main Results

Let us consider the price $S(t)$ of a financial object (an index, an exchange rate, a share quote) at time t, and define the logarithmic price as $x(t) \equiv \ln S(t)$. A moving average $\bar{x}(t)$ based on logarithmic prices, that takes into account all the past with an exponentially time decaying weight, can be written in the form

$$\bar{x}(t) = (1-\beta)\sum_{n=0}^{\infty}\beta^n x(t-n-1)\,. \tag{1}$$

The moving average at time t is computed only with past quotes with respect to t, and $x(t)$ is not included. This is not a relevant choice, since all the following could be reformulated in an equivalent way, including $x(t)$ in the moving average. The parameter $0 < \beta < 1$ controls the memory length: for $\beta = 0$ we have the shortest (one step) memory $\bar{x}(t) = x(t-1)$, while when β approaches 1 the memory becomes infinity and flat (all the past prices have the same weight). More precisely,

β determines the typical past time scale $1-\ln 2/\ln\beta$ up to that we have a significant contribution in the average (1).

Let us suppose that future price linearly depends on the difference between the current price and the moving average, plus a certain degree of randomness. In our model $x(t+1)$ is written as

$$x(t+1) = x(t) + \alpha[x(t) - \bar{x}(t)] + \sigma\omega(t)\,, \tag{2}$$

where the $\omega(t)$ at varying t are a set of independent identically distributed random variables, with vanishing mean and unitary variance, so that $\sigma\omega(t)$ is a random variable of variance σ^2. Indeed, our analysis focuses on the problem of correlations with moving averages and we expect that results would not qualitatively depend on the shape of ω distribution, which likely is a fat tails distribution [11–13].

The parameter $-1 < \alpha < +1$ adjusts the impact of the difference $x(t) - \bar{x}(t)$ over the future price. Notice that a positive α means that the moving average is repulsive (the price most likely has a positive change if $x(t)$ is larger then $\bar{x}(t)$), on the contrary the moving average is attractive if α is negative. Also notice that the process loses all memory from the past for $\alpha = 0$, independently on β, becoming a pure random walk. Let us stress that the Vasicek model for interest rates [6] is a particular case in our more general scheme: it corresponds to the continuous time version of the peculiar choice $\beta = 1$, where $\omega(t)$ is a zero mean gaussian variable.

According to (2), the return $r(t)$, defined as $r(t) \equiv \ln(S(t+1)/S(t))$, turns out to be $r(t) = \alpha[x(t) - \bar{x}(t)] + \sigma\omega(t)$. Let us stress that the first contribution is measurable at time t just before the price change and, therefore, is an available information for the trader.

It is easy to check that the moving average (1) satisfies

$$\bar{x}(t+1) = (1-\beta)x(t) + \beta\bar{x}(t)\,. \tag{3}$$

Equations (2) and (3) define a two-component linear Markov process. Such a process can be solved by a diagonalization procedure. After having defined $\tilde{\alpha} \equiv \frac{\alpha}{1-\beta}$ and $\epsilon \equiv \alpha + \beta$, we introduce a new couple of variables, linear combinations of $x(t)$ and $\bar{x}(t)$

$$\begin{aligned} y(t) &\equiv x(t) - \bar{x}(t)\,, \\ z(t) &\equiv x(t) - \tilde{\alpha}\bar{x}(t)\,, \end{aligned} \tag{4}$$

being $y(t)$ simply the difference between the logarithmic price and its moving average at time t. The system turns out to be diagonal in terms of the new variables

$$\begin{aligned} y(t+1) &= \epsilon y(t) + \sigma\omega(t)\,, \\ z(t+1) &= z(t) + \sigma\omega(t)\,, \end{aligned}$$

and therefore can be easily solved, obtaining

$$y(t) = \sigma \sum_{n=0}^{t-1} \epsilon^n \omega(t-n-1) + \epsilon^t y(0)\,,$$
$$z(t) = \sigma \sum_{n=0}^{t-1} \omega(t-n-1) + z(0)\,. \tag{5}$$

The steady distribution $p_{\tilde{\sigma}}(y)$ of y can be derived from equation (5), once given the ω distribution. The variance of y, $\tilde{\sigma}^2$, can be directly computed

$$\tilde{\sigma}^2 \equiv \langle y^2 \rangle = \frac{\sigma^2}{1-\epsilon^2}\,,$$

where $\langle . \rangle$ means average over the ω distribution. Let us stress that the parameter ϵ must satisfy $|\epsilon| < 1$ in order to prevent a negative $\tilde{\sigma}^2$ or diverging quantities in the solution (5).

The solution in terms of $x(t)$ and $\bar{x}(t)$ can be obtained from (5) by using the inverse transformation of (4)

$$x(t) = x(0) + \frac{1-\beta}{1-\epsilon}\left[\sigma \sum_{n=0}^{t-1}(1-\tilde{\alpha}\epsilon^n)\omega(t-n-1) + \tilde{\alpha}(1-\epsilon^t)[x(0)-\bar{x}(0)]\right],$$

$$\bar{x}(t) = \bar{x}(0) + \frac{1-\beta}{1-\epsilon}\left[\sigma \sum_{n=0}^{t-1}(1-\epsilon^n)\omega(t-n-1) + (1-\epsilon^t)[x(0)-\bar{x}(0)]\right].$$

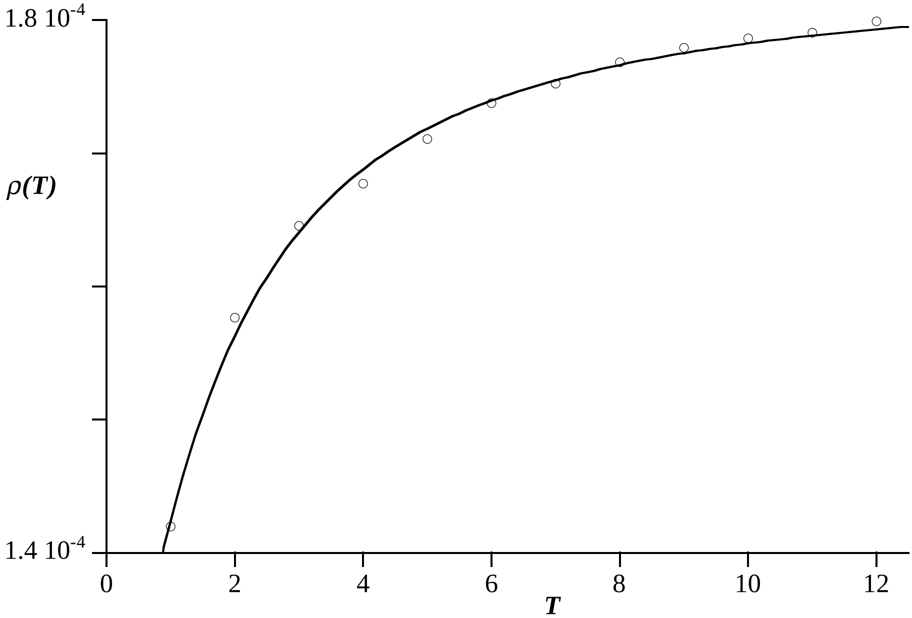
In order to test our model against real financial data, we need to find a measurable quantity that can be computed without specifying the ω distribution. For this reason, a direct comparison of the logarithmic prices $x(t)$ with financial datasets cannot be of some help. On the contrary the variance of any time window is independent on the details of ω distribution. It is, therefore, intuitive to consider $\rho(T)$, the T-time steps quadratic volatility scaled with T, defined as

$$\rho(T) \equiv \frac{1}{T}\langle [x(t+T) - x(t)]^2 \rangle\,.$$

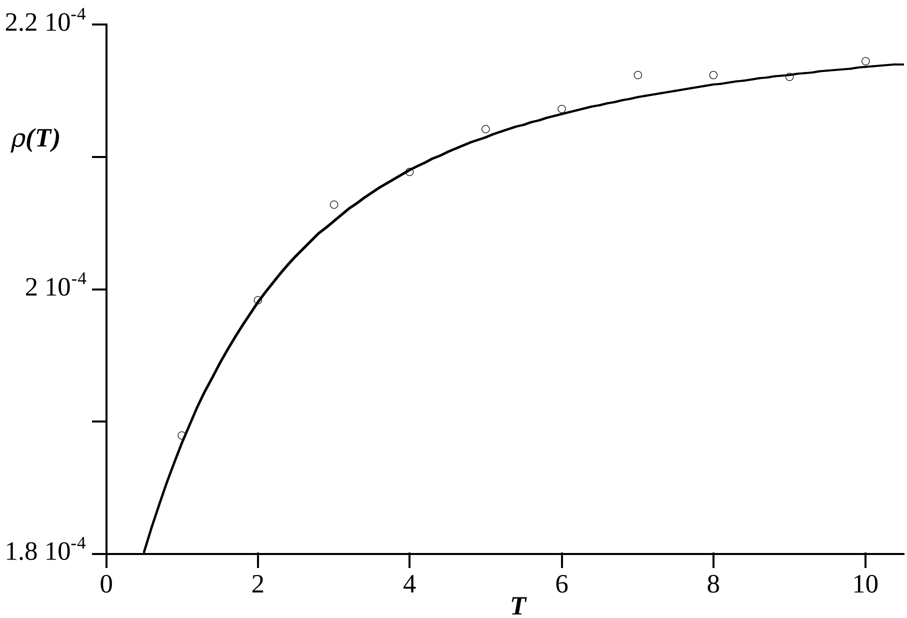
In the context of our model, the above quantity is

$$\rho(T) = \frac{\sigma^2}{T}\frac{(1-\beta)^2}{(1-\epsilon)^2}\left[T - \frac{2\tilde{\alpha}(1+\epsilon-\tilde{\alpha})}{1+\epsilon}\frac{1-\epsilon^T}{1-\epsilon}\right]. \tag{6}$$

Notice that for $\alpha = 0$ this quantity would be a constant, while it increases with T when α is positive (repulsive moving averages), and it decreases with T when α is negative (attractive moving averages). Notice also that the one-step ($T = 1$) quadratic volatility is $\rho(1) = \sigma^2(1-\epsilon^2+\alpha^2)/(1-\epsilon^2)$, which is always larger than σ^2. In other terms, the one-step volatility is not simply σ, the component due to the random variables, but it is systematically larger due to the influence of the moving average. In the long range ($T \to \infty$) $\rho(T)$ tends to $\sigma^2(1-\beta)^2/(1-\epsilon)^2$, which is larger (smaller) then σ^2 for positive (negative) α.

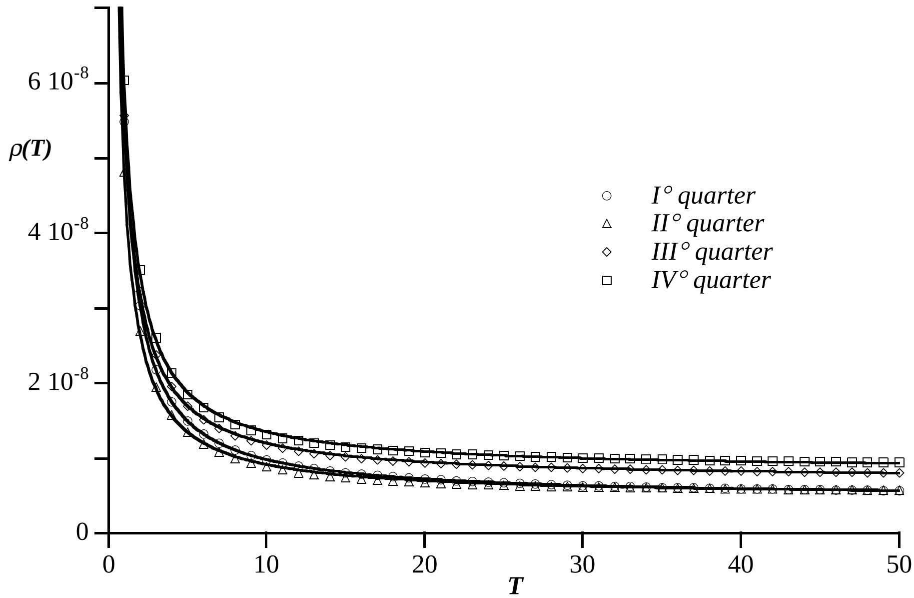


(a)



(b)

Fig. 1. T-time steps quadratic volatility scaled with T, $\rho(T)$ (circles), as a function of T, for: (a) Nasdaq index; (b) Mibtel index; (c) 1998 USD–DM high frequency exchange rate. The continuous line represents the best fit of expression (6).



(c)

Fig. 1. (*Continued*)

The expression (6) of $\rho(T)$ can be fitted with real data varying the free parameters α, β and σ. In Fig. 1 $\rho(T)$ is plotted for three datasets: Nasdaq index (Fig. 1(a)), Mibtel index (Fig. 1(b)) and 1998 USD–DM high frequency exchange rate (Fig. 1(c)).

In the first two cases the agreement between data and expression (6) predicted by our model is excellent up to a critical value of T, which depends on the size of the dataset. For T larger than the critical value insufficient statistics makes the experimental values no longer significant.

In particular, we have obtained the following results:

(a) Nasdaq index: $\alpha = (9.51 \pm 0.01) \cdot 10^{-2}$, $\beta = (2.49 \pm 0.01) \cdot 10^{-1}$ and $\sigma = (1.187 \pm 0.001) \cdot 10^{-2}$ in the range $1 \leq T \leq 12$;
(b) Mibtel index: $\alpha = (5.54 \pm 0.01) \cdot 10^{-2}$, $\beta = (3.15 \pm 0.01) \cdot 10^{-1}$ and $\sigma = (1.370 \pm 0.001) \cdot 10^{-2}$ in the range $1 \leq T \leq 10$.

The two daily indices not only behave qualitatively the same, but also exhibit similar parameters. The main result is that they have a positive α, which means that the moving average repulses the spot price. Moreover, in both cases the typical memory length is about 2 time steps, which corresponds to a couple of trading days, and σ, which is nearly the daily market volatility, turns out to be about 1% per day. Let us stress that this is only the random component of daily volatility, this last being larger because of the deterministic component contribution.

By virtue of its high number of data, the 1998 USD–DM high frequency exchange rate case permits us to test the stability in time of the parameters of the model. For this reason we have divided the whole year into quarters of three months, and then we have performed an independent fit of (6) for each quarter. In Fig. 1(c) are plotted the function $\rho(T)$ computed from the dataset and the respective fit (always performed in the range $1 \leq T \leq 100$) for each quarter.

The numerical results are:

(c) 1998 USD–DM exchange rate

(i) quarter (January–March 1998): $\alpha = (-6.37 \pm 0.01) \cdot 10^{-1}$, $\beta = (6.61 \pm 0.01) \cdot 10^{-1}$ and $\sigma = (1.976 \pm 0.001) \cdot 10^{-4}$;
(ii) quarter (April–June 1998): $\alpha = (-6.25 \pm 0.01) \cdot 10^{-1}$, $\beta = (6.27 \pm 0.01) \cdot 10^{-1}$ and $\sigma = (1.862 \pm 0.001) \cdot 10^{-4}$;
(iii) quarter (July–September 1998): $\alpha = (-5.76 \pm 0.01) \cdot 10^{-1}$, $\beta = (6.00 \pm 0.01) \cdot 10^{-1}$ and $\sigma = (2.046 \pm 0.001) \cdot 10^{-4}$;
(iv) quarter (October–December 1998): $\alpha = (-5.68 \pm 0.01) \cdot 10^{-1}$, $\beta = (5.79 \pm 0.01) \cdot 10^{-1}$ and $\sigma = (2.137 \pm 0.001) \cdot 10^{-4}$.

The USD–DM high frequency exchange rate has a negative α, which means that the moving average attracts the spot price. Moreover it exhibits a more consistent influence of the moving average with respect to the indices, since $|\alpha|$ is about one order of magnitude larger, and the typical memory turns out to be about 2–3 time steps (about 1 minute).

The agreement between data and respective fit is remarkable for each individual quarter, and these results fully support the consistence of the proposed model. Nevertheless the parameters are not stable among different quarters: in fact in the 1998 USD–DM case α, β and σ exhibit fluctuations of about 10%. It is quite interesting to note that during 1998 a decrease of the attractive power of the moving average $|\alpha|$ and a reduced typical memory of the process (via β) correspond to an increment of the random volatility σ. These facts suggests that the parameters instability during 1998 is not due to random fluctuations, but could be the result of small structural changes of foreign exchange market, consisting in an increment of the intrinsic random volatility that has reduced the influence of moving average on price dynamics.

3. Conclusion

Moving average plays an important role in price dynamics. Future price is influenced by the current difference between logarithmic price and its moving average. This difference tends both to reduce and to enlarge, according to the examined market. The first open question is: this feature is a strict peculiarity of the market, or does it depends on the time frequency of data? In our cases, both daily datasets exhibit a repulsive moving average, at difference with USD–DM high frequency dataset.

The answer could be that at different time scales, different moving average effects are active, possibly attractive on short scales and repulsive on larger scales. The problem deserves further investigations.

Another interesting point is to investigate if the moving average action is generated by a self-organized mechanism of traders reactions [15, 16] In other words, is it possible that traders, taking into account informations given by moving averages, make collectively induced financial choices producing, as a result, the observed phenomena?

In our price dynamics model a random component is also present, which we do not have deeply investigated, being this out of the scope of this letter. Nevertheless, our picture could help to determine the exact shape of the noise, since, in principle, we are now able to filter the deterministic contribution.

Acknowledgmnts

We thank Dietrich Stauffer for many useful discussions and for a critical reading of the manuscript. R. B. thanks the European Community TMR Grant: Financial Market Efficiency and Economic Efficiency.

References

[1] M. J. Pring, *Technical Analysis Explained* (McGraw-Hill, 1985).
[2] T. A. Meyers, *The Technical Analysis Course* (Probus Publishing, 1989).
[3] M. Ratner and R. P. Leal, Test of technical trading strategies in the emerging equity markets of Latin America and Asia, *J. Banking and Finance* **23** (1999) 1887–1905.
[4] K. Ojah and D. Karemera, Random walks and market efficiency tests of Latin Amaeracan emerging equity markets: A revisit, *The Financial Review* **34** (1999) 57–72.
[5] D. Meyers, Making money from memories, *Futures* **29** (2000) 60–62.
[6] A. M. R. Taylor, Testing for quasi-market forces in secondary education, *Oxford Bulletin of Economics and Statistics* **62** (2000) 291–324.
[7] F. R. Johnston, J. E. Boyland, M. Meadows and E. Shale, Some properties of a simple moving average when applied to forecasting a time series, *J. Operational Res. Soc.* **50** (1999) 1267–1271.
[8] R. N. Mantegna and H. E. Stanley, Turbulence and financial markets, *Nature* **383** (1996) 587–588.
[9] R. N. Mantegna and H. E. Stanley, *An Introduction to Econophysics: Correlations and Complexity in Finance* (Cambridge University Press, 1999).
[10] M. Pasquini and M. Serva, Multiscale behaviour of volatility autocorrelations in a financial market, *Economics Letters* **65** (1999) 275–279.
[11] P. K. Clark, A subordinated stochastic process model with finite variance for speculative prices, *Econometrica* **41** (1973) 135–155.
[12] B. B. Mandelbrot, The variation of certain speculative prices, *J. Business* **36** (1963) 394–419.
[13] R. N. Mantegna and H. E. Stanley, Scaling behaviour in the dynamics of an economic index, *Nature* **376** (1995) 46–49.
[14] O. Vasicek, An equilibrium characterization of the term structure, *J. Financial Econ.* **5** (1977) 177–188.

[15] T. Halpin-Healy and Y. C. Zhang, Kinetic Roughening, Stochastic Growth, Directed Polymers and all that, *Phys. Rep.* **254** (1995) 215–415.

[16] T. Palmer, *Forecasting Financial Markets* (Kagan Page, 1993).